

SM3 7.2 Log Evaluations & Properties

Exponential expressions are of the form a^x and have the value of multiplying $a \cdot a \cdot a \cdot a \cdot a \cdots a$ until a total of x copies of a have been multiplied together. If x is negative, we divide by x copies of a instead.

Example: Evaluate 2^4

$$2 \cdot 2 \cdot 2 \cdot 2$$

$$16$$

Example: Evaluate 3^{-4}

$$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$$

$$\frac{1}{81}$$

Logarithmic expressions are of the form $\log_b a$ where b represents the logarithmic base and a represents the argument of the logarithm. The expression has a value that answers the question "how many b s need to be multiplied to produce a ?"

Example: Evaluate $\log_3 81$

81 is a power of 3

Since $3 \cdot 3 \cdot 3 \cdot 3 = 81$, we need to multiply by 4 copies of 3 to get 81.

$$\log_3 81 = 4$$

Example: Evaluate $\log_2 \frac{1}{16}$

16 is a power of 2 but since it's in the bottom, we'll need negative power.

Since $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$, we need to divide by 4 copies of 2 to get $\frac{1}{16}$.

$$\log_2 \frac{1}{16} = -4$$

Logarithmic and exponential operations are inverses: they undo one another. If you have to evaluate a base raised to a like-based log, the two operations cancel. Likewise, when evaluating the log of an exponential that shares the same base, the two operations cancel.

Example: Evaluate $\log_3 3^7$

$\log_3 x$ and 3^x are inverse operations and eliminate one another.

$$\log_3 3^7 = 7$$

Example: Evaluate $6^{\log_6 11}$

6^x and $\log_6 x$ are inverse operations and eliminate one another.

$$6^{\log_6 11} = 11$$

Since logarithms and exponentials are related then we can switch between their forms. We know that $3^2 = 9$ and $\log_3 9 = 2$ (because 2 copies of 3's are needed to make 9), this leads us to see that we can switch between exponential form and logarithmic if needed.

$$\begin{array}{ccc} \text{Exponential Form} & \leftrightarrow & \text{Logarithmic Form} \\ b^c = a & & \log_b a = c \end{array}$$

Evaluate the following expressions

1. 3^5

2. 4^{-2}

3. 10^3

4. -2^{-2}

5. $\log_3 27$

6. $\log_4 4$

7. $\log_8 1$

8. $\log_3 \frac{1}{81}$

9. $\log_5 125$

10. $\log_{12} 144$

11. $\log 1000$

12. $\log 0.001$

13. $2^{\log_2 12}$

14. $\log_{11} 11^{-3}$

15. $\log_4 16^x$

16. $\log_2 \frac{1}{32}$

17. $6^{\log_6(2x+1)}$

18. $\log_6 \frac{1}{216}$

19. $\log_8 8^7$

20. $\log_{16} 4$

21. e^0

22. $\ln 1$

23. $e^{\ln x}$

24. $\ln(e)^2$

Rewrite each exponential in logarithmic form.

25. $81^{1/2} = 9$

26. $19^2 = 361$

27. $\frac{1}{32} = 2^{-5}$

28. $r^8 = 117$

Rewrite each logarithm in exponential form.

29. $\log_{12} \frac{1}{144} = -2$

30. $\log_{15} 225 = 2$

31. $\log_{11} y = x$

32. $\log_6 1 =$

Complete the tables of values of a function:

| 33) $f(x) = \log_3 x$ | |
|-----------------------|--------|
| x | $f(x)$ |
| $\frac{1}{9}$ | |
| $\frac{1}{3}$ | |
| 1 | |
| 3 | |
| 9 | |

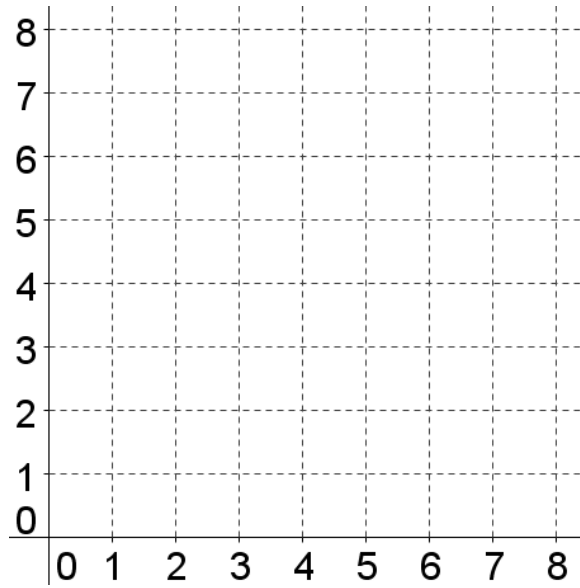
| 34) $g(x) = \log_2 x$ | |
|-----------------------|--------|
| x | $g(x)$ |
| 1 | |
| 16 | |
| $\frac{1}{8}$ | |
| 1024 | |
| $\frac{1}{32}$ | |

| 35) $p(x) = \log_5 x$ | |
|-----------------------|---------------|
| x | $p(x)$ |
| | 2 |
| | 0 |
| | -3 |
| | 4 |
| | $\frac{1}{2}$ |

| 36) $q(x) = \log x$ | |
|---------------------|--------|
| x | $q(x)$ |
| | 4 |
| | -1 |
| | 0 |
| | 6 |
| | -3 |

37) Complete the tables then graph both functions on the same coordinate axis by plotting points and connecting with a curve.

| $v(x) = 2^x$ | | $w(x) = \log_2 x$ | |
|--------------|--------|-------------------|--------|
| x | $v(x)$ | x | $w(x)$ |
| 1 | | | 1 |
| 2 | | | 2 |
| 3 | | | 3 |



| $v(x) = e^x$ | | $w(x) = \ln x$ | |
|--------------|--------|----------------|--------|
| x | $v(x)$ | x | $w(x)$ |
| 1 | | | 1 |
| 2 | | | 2 |
| 3 | | | 3 |

